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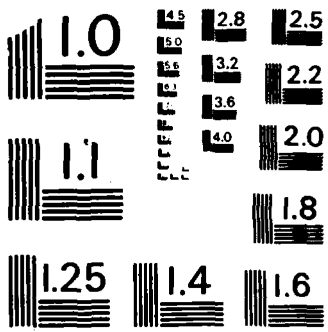

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TECHNICAL REPORT RD-GC-86-1

EVALUATION OF THE MULTIFUNCTION SENSOR ACCELERATION SENSITIVE TERMS MU_A AND MU_B

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OCTOBER 1985

U.S. ARMY MISSILE COMMAND
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bias	axes orientations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Collins makes the assumption that g-sensitive terms MU_{A2} and MU_{B2} are small and therefore negligible. The purpose of this study is to determine if this assumption is valid. Laboratory tests were carefully performed to confirm the magnitudes of these terms. The results clearly show a significant gyro error when the terms are assumed small, stable, and negligible.

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I. INTRODUCTION

The purpose of this report is to determine the validity of Rockwell Collins' assumptions that g-sensitive terms MU_{A2} and MU_{B2} are small, stable, and negligible. This report contains the results of tests and evaluations conducted by the Research, Development, and Engineering Center to confirm the magnitudes of these terms. Appendix A contains a detailed discussion of tests and test results.

II. BACKGROUND

Rockwell Collins assumes that the magnitude of the gyro acceleration sensitive terms MU_{A2} and MU_{B2} in equations (1) through (6) below are small, stable, and negligible. It is well known that the GSA_2 and GSB_2 acceleration sensitive terms in these equations are highly temperature dependent.

The Collins Phase III Multifunction Sensor Inertial Measurement Unit (IMU) is used in the evaluation. Prelaunch self-calibration measurements are made at three stationary orientations. These orientations for gyro 2, at each measurement position, are shown in Figure 1. Figure 1 also provides the components of acceleration (G_1, G_2, G_3). The measurement equations for each gyro output data axis are as follows:

The measurement equations for Collect 1 orientation are:

$$RA_2{}^1 = B_{A2} - GSA_2 * G_1 + GSB_2 * G_2 + MU_{B2} * G_3 - W_1 \qquad (1)$$

$$RB_2{}^1 = B_{B2} + GSA_2 * G_2 + GSB_2 * G_1 - MU_{A2} * G_3 + W_2 \qquad (2)$$

Rotation of the multisensor to Collect 2 position yields the following static measurement equations:

$$RA_2{}^2 = B_{A2} + GSA_2 * G_1 + GSB_2 * G_2 - MU_{B2} * G_3 + W_1 \qquad (3)$$

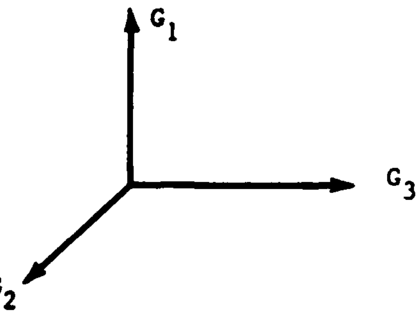
$$RB_2{}^2 = B_{B2} + GSA_2 * G_2 - GSB_2 * G_1 + MU_{A2} * G_3 + W_2 \qquad (4)$$

Rotation of the multisensor to Collect 3 position yields the following static measurement equations:

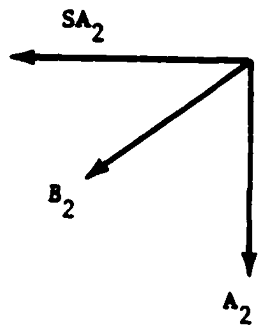
$$RA_2{}^3 = B_{A2} + GSA_2 * G_3 + GSB_2 * G_2 + MU_{B2} * G_1 + W_3 \qquad (5)$$

$$RB_2{}^3 = B_{B2} + GSA_2 * G_2 - GSB_2 * G_3 - MU_{A2} * G_1 + W_2 \qquad (6)$$

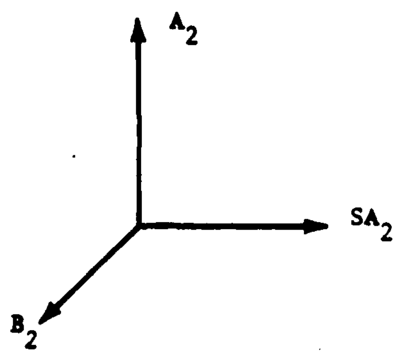
Acceleration Components



Collect 1



Collect 2



Collect 3
(Navigation Orientation)

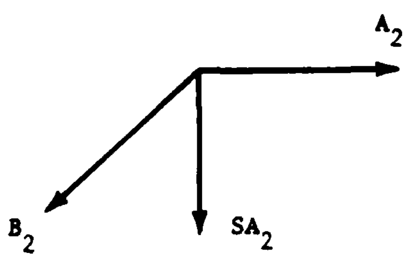


Figure 1. Axes orientation for data collection.

The six measurement equations above are used to solve for g-sensitive (GSA_2, GSB_2, MU_{A2}, MU_{B2}) and bias (B_{A2}, B_{B2}) terms. The on-axis g-sensitive (GSA_2) term is obtained by subtracting equation (1) from equation (3).

$$GSA_2 * G_1 = (RA_2{}^2 - RA_2{}^1) \div 2 + MU_{B2} * G_3 - W_1 \qquad (7)$$

The cross-axis g-sensitive (GSB_2) term is obtained by subtracting equation (4) from equation (2):

$$GSB_2 * G_1 = (RB_2{}^1 - RB_2{}^2) \div 2 + MU_{A2} * G_3 \qquad (8)$$

Observe that the stability of the GSA_2 and GSB_2 terms depend on the validity of the MU_{B2} and MU_{A2} assumptions and how well the acceleration components are defined. For this evaluation, the magnitudes of the G_2 and G_3 gravity components are reduced to approximately zero.

Equations (9) and (10), derived from equations (1) through (4), are used to compute the gyro bias (B):

$$B_{A2} = (RA_2{}^2 + RA_2{}^1) \div 2 - GSB_2 * G_2 \qquad (9)$$

$$B_{B2} = (RB_2{}^2 + RB_2{}^1) \div 2 - GSA_2 * G_2 - W_2 \qquad (10)$$

Note the bias stability is dependent on the predictability of the GSA_2, GSB_2, and G_2 terms.

To calculate sensor 2 MU_{A2} and MU_{B2} terms, sensor 2 gyro equations were set up in the following matrix form:

$$\begin{bmatrix} RA_2\ (C1) + W_1 \\ RB_2\ (C1) - W_2 \\ RA_2\ (C2) - W_1 \\ RB_2\ (C2) - W_2 \\ RA_2\ (C3) - W_3 \\ RA_2\ (C3) - W_2 \end{bmatrix} = \begin{bmatrix} 0 & g_3 & 1 & 0 & -g_1 & g_2 \\ -g_3 & 0 & 0 & 1 & g_2 & g_1 \\ 0 & -g_3 & 1 & 0 & g_1 & g_2 \\ g_3 & 0 & 0 & 1 & g_2 & -g_1 \\ 0 & g_1 & 1 & 0 & g_3 & g_2 \\ -g_1 & 0 & 0 & 1 & g_2 & -g_3 \end{bmatrix} \begin{bmatrix} MU_{A2} \\ MU_{B2} \\ B_{A2} \\ B_{B2} \\ GSA_2 \\ MU_{SA2} \end{bmatrix}$$

The following augmented matrix was obtained, row reduction procedures applied, and the solution to the six gyro parameters found.

The sensor 2 augmented matrix is

$$\begin{bmatrix} 0 & g_3 & 1 & 0 & -g_1 & g_2 & C_1 \\ -g_3 & 0 & 0 & 1 & g_2 & g_1 & C_2 \\ 0 & -g_3 & 1 & 0 & g_1 & g_2 & C_3 \\ g_3 & 0 & 0 & 1 & g_2 & -g_1 & C_4 \\ 0 & g_1 & 1 & 0 & g_3 & g_2 & C_5 \\ -g_1 & 0 & 0 & 1 & g_2 & -g_3 & C_6 \end{bmatrix}$$

where

$$C_1 = RA_2\ (C1) + W_1$$

$$C_2 = RB_2\ (C1) - W_2$$

$$C_3 = RA_2\ (C2) - W_1$$

$$C_4 = RB_2\ (C2) - W_2$$

$$C_5 = RA_2\ (C3) - W_3$$

$$C_6 = RB_2\ (C3) - W_2 \quad .$$

The sensor 2 matrix gyro parameter solutions for MU_{A2} and MU_{B2} are as follows:

$$MU_{A2} = \frac{(RB_2{}^1 + RB_2{}^2 - 2RB_2{}^3)\ G_1 + (RB_2{}^2 - RB_2{}^1)\ G_3}{2(G_1{}^2 + G_3{}^2)} \qquad (11)$$

and

$$MU_{B2} = \frac{(2RA_2^{\,3} - RA_2^{\,1} - RA_2^{\,2} - 2W_3)\ G_1 + (RA_2^{\,1} - RA_2^{\,2} + 2W_1)\ \ G_3}{2(G_1^{\,2} + G_3^{\,2})} \qquad (12)$$

III. EVALUATION OF THE MU_{A2} AND MU_{B2} ACCELERATION TERMS

During these evaluations, an environmental chamber, placed over the multifunction IMU, provided the mechanism for controlling the ambient temperature environment. Twenty-four prelaunch self-calibration tests were made with the instrument mounted in the navigation orientation. Three self-calibration tests were made at each ambient temperature setting. Forty-five minutes between each self-calibration tests were allowed for the instrument to thermally restabilize. The sequence was repeated at the eight designated temperature settings.

The test results, showing the computed mean magnitude values and one sigma randomness of the gyro bias, MU_{A2}, and MU_{B2} drifts, are summarized in Table 1. Figures 2 and 3 show line graphs of the bias data plotted for the eight temperature settings. The matrix line graphs (M) (Figs. 2 and 3) represent the bias drift obtained by using equations (9) and (10). In utilizing equations (9) and (10), the MU_{A2} and MU_{B2} acceleration-sensitive terms cancel and the bias is dependent only on the predictability of the $GSA_2 * G_2$ and $GSB_2 * G_2$ terms. Line graphs, derived from equations (5) and (6) which assume the magnitudes of MU_{A2} and MU_{B2} terms are zero, are plotted also in Figures 2 and 3 with the appropriate matrix line graph.

The line graphs (Figs. 4 and 5) depict the magnitudes of the MU_{A2} and MU_{B2} acceleration sensitive terms derived by using equations (11) and (12). These line graphs represent the terms that Collins assumed were small, stable and negligible.

TABLE 1. Results from Temperature Tests

Gyro 2 Parameters

Temperature	-21.5 °F	6.1 °F	+19.7 °F	43.3 °F	74.1 °F	100.1 °F	120 °F	140 °F
MU_{A2} Matrix (°/hr/g)	-37.2333 ±7.3546	4.2000 ±1.8974	0.6500 ±10.2736	23.8533 ±0.7411	9.1933 ±4.8964	6.0640 ±4.3347	-0.4467 ±0.6108	1.9767 ±1.4003
MU_{B2} Matrix (°/hr/g)	-7.5267 ±0.9765	3.6433 ±1.7180	-24.3367 ±6.5150	-18.3400 ±6.3700	0.0500 ±1.1053	3.3700 ±3.8234	-1.6433 ±5.7097	-0.2600 ±1.3647
RA_2 Bias COLLINS (°/hr)	14.4833 ±3.2739	31.2800 ±3.8800	-2.8333 ±8.0816	4.6833 ±6.9513	17.7233 ±0.6000	13.0140 ±2.7846	4.4300 ±6.1035	11.1733 ±1.0719
RA_2 Bias Matrix (°/hr)	22.0800 ±3.9131	30.5400 ±1.4388	21.4867 ±1.7114	23.0500 ±1.6985	17.7333 ±0.8406	9.6740 ±1.6794	6.0700 ±1.6982	11.4267 ±0.4708
RB_2 Bias COLLINS (°/hr)	-0.5833 ±4.6047	-18.1167 ±2.3702	-28.2300 ±21.2988	-28.7933 ±3.0643	10.2667 ±4.7007	20.9220 ±9.3362	34.5567 ±0.5554	29.2433 ±1.7993
RB_2 Bias Matrix (°/hr)	-37.8267 ±2.7822	-13.7500 ±3.0541	-27.7167 ±14.3048	-5.1667 ±3.4817	19.4900 ±1.0224	26.9940 ±7.7122	34.7000 ±0.6791	31.2000 ±0.6710

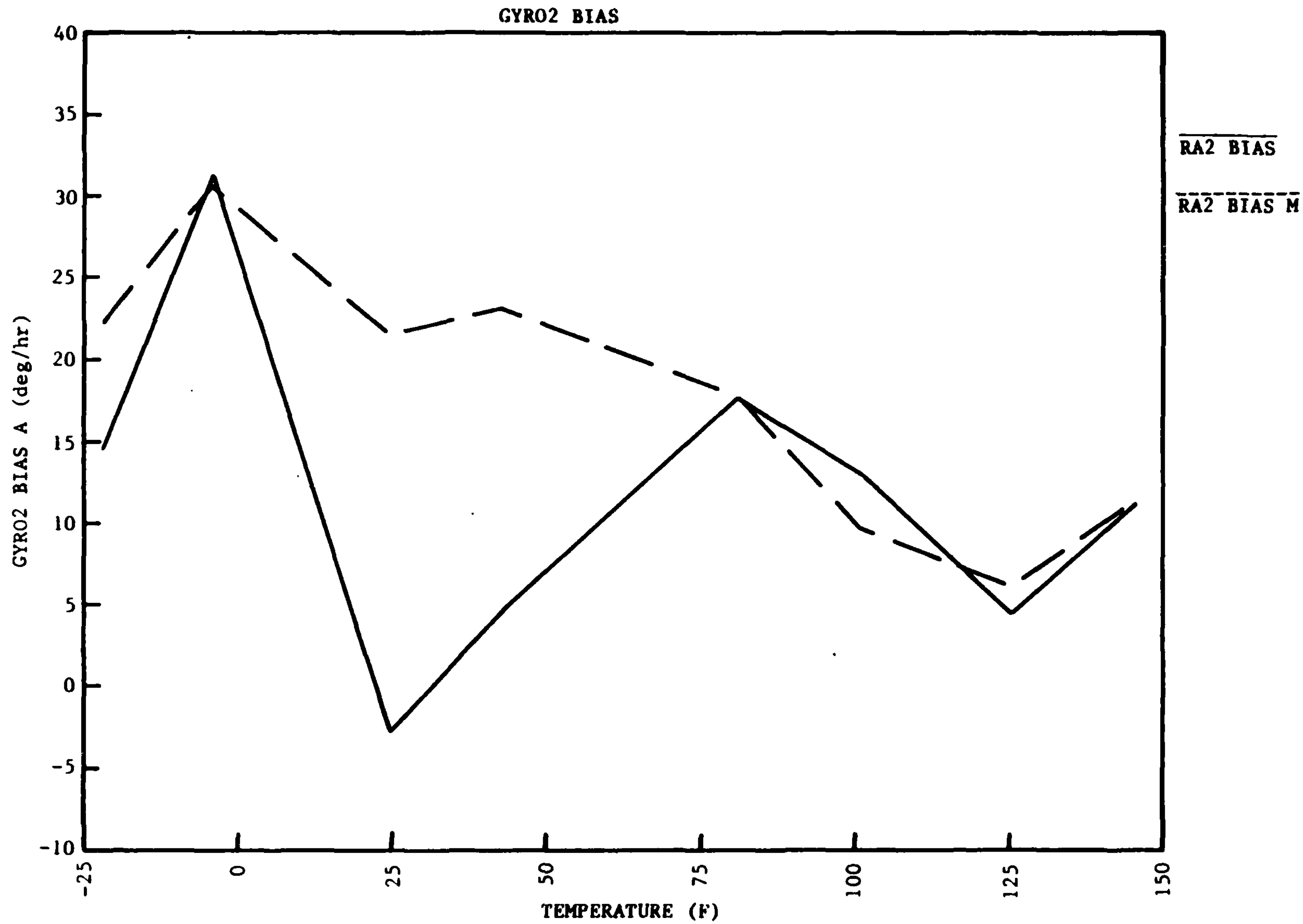


Figure 2. Bias A drift vs temperature.

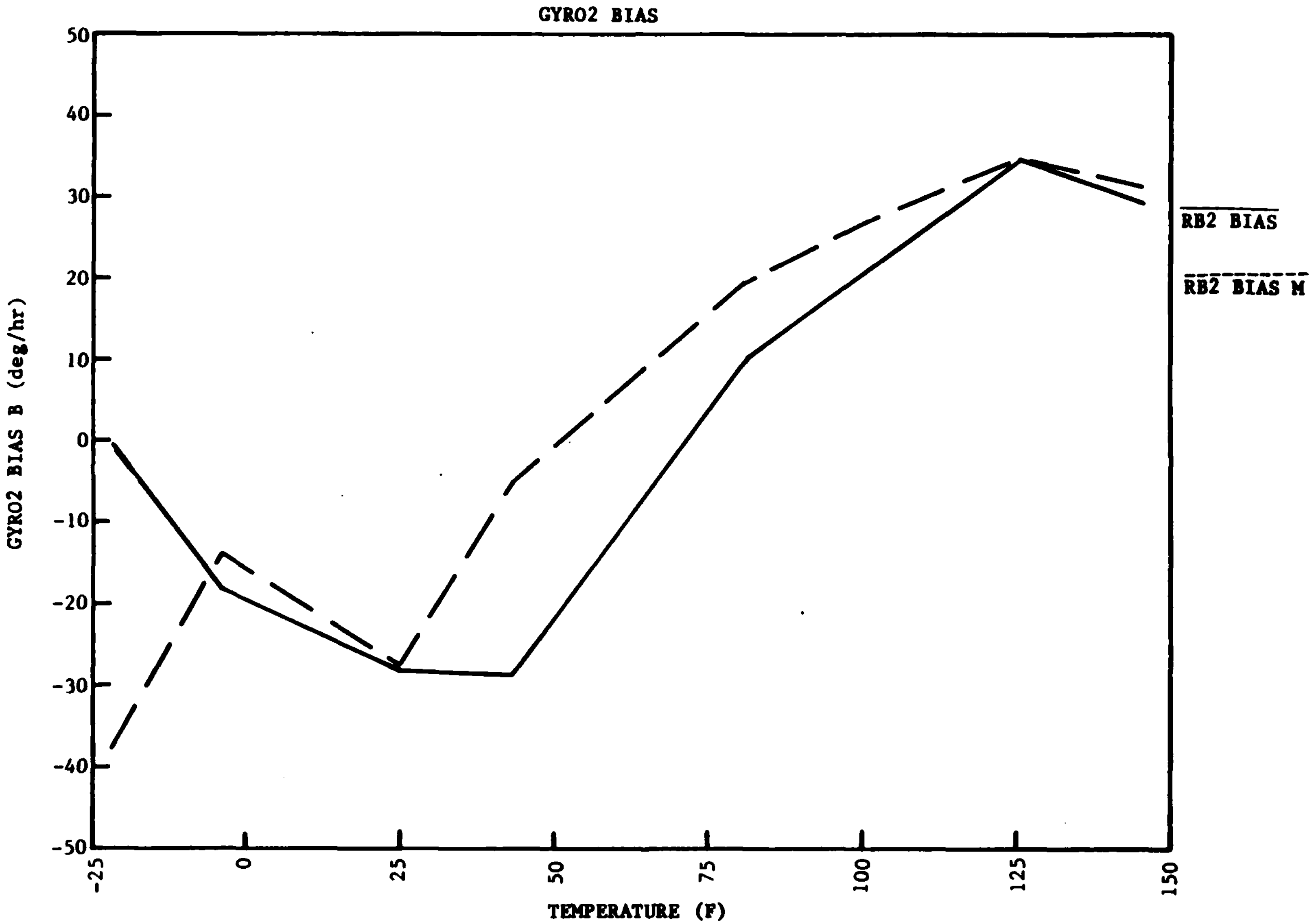


Figure 3. Bias B drift vs temperature.

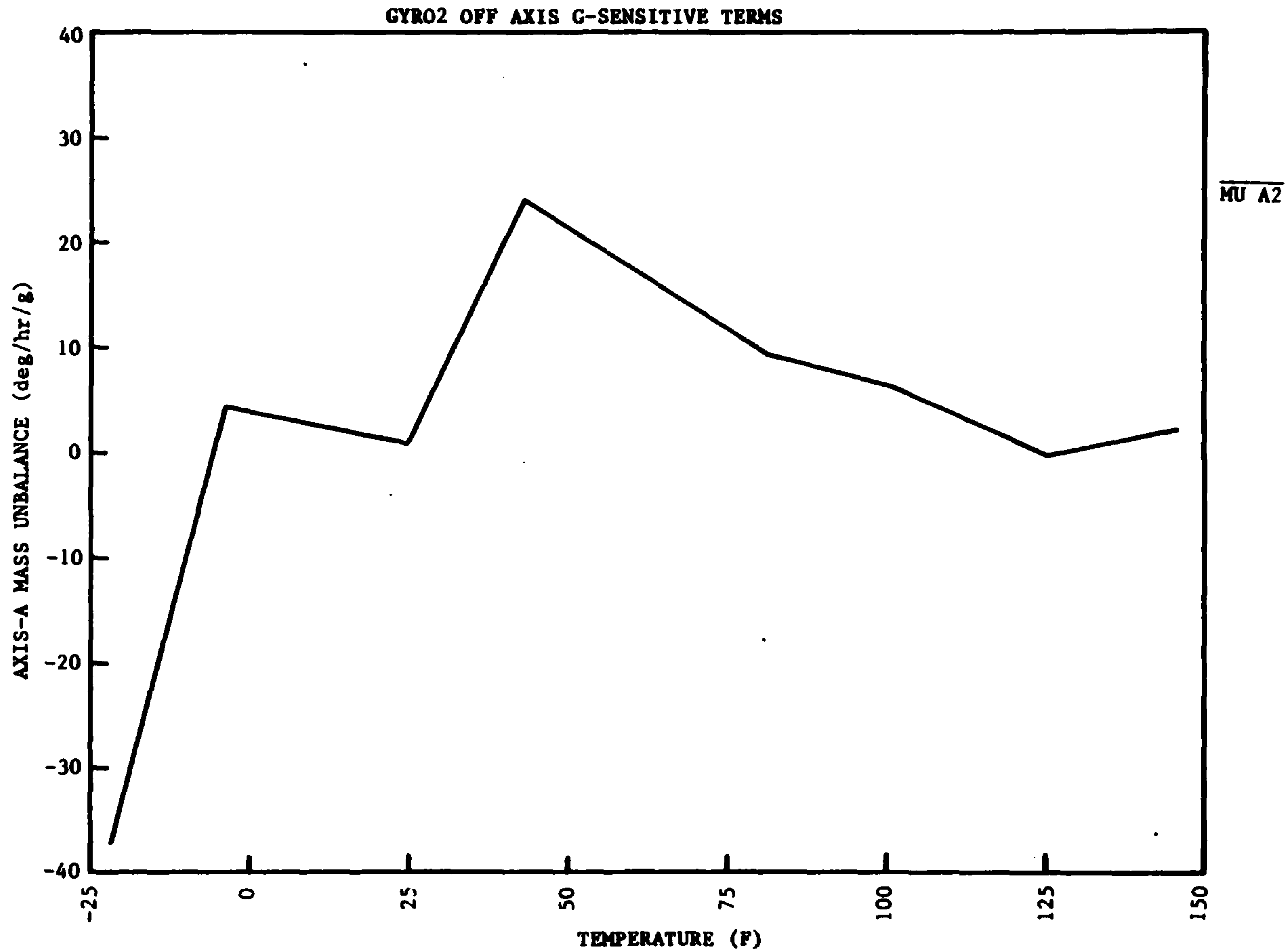


Figure 4. MU_A drift vs temperature.

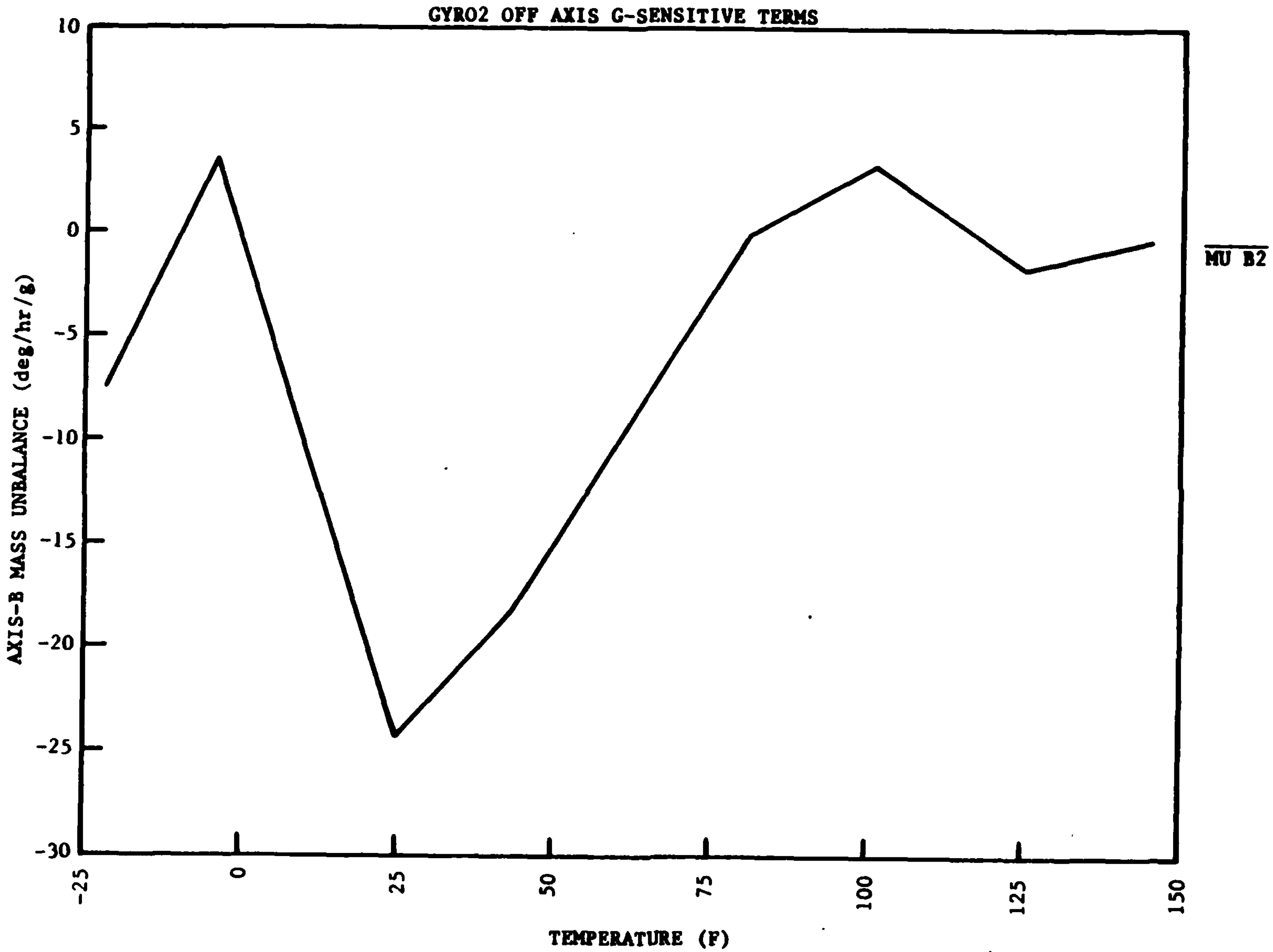


Figure 5. MU_B drift vs temperature.

IV. CONCLUSIONS

From the tests and evaluations conducted, data were obtained (Table 1) which confirms that the MU_{A2} and MU_{B2} acceleration sensitive terms are temperature dependent. The magnitude of the MU_A term varies from 23.8 to -37.2°/hr/g with a maximum uncertainty of 10.3°/hr/g one-sigma. The magnitude of the MU_B term varies from 3.6 to -24.3°/hr/g with a maximum uncertainty of 6.4°/hr/g one-sigma. In conclusion, the bias drift data also reflects the impact of assuming that MU_{A2} and MU_{B2} terms are equal to zero. The line graphs (Figs. 2 and 3) clearly illustrate significant differences in the bias drift magnitudes when the terms MU_{A2} and MU_{B2} are assumed small, stable, and negligible when they are not.

V. RECOMMENDATIONS

The MU_{A2} and MU_{B2} acceleration/temperature sensitive drift rates do play a significant role in computing the gyro bias if equations (5) and (6) are used. To eliminate these terms, it is recommended that bias equations (9) and (10) be used.

In using equations (9) and (10), the computed bias values are still dependent on the predictability of the following terms: GSA_2 and GSB_2 acceleration/temperature sensitive drifts, the acceleration components defined by the multisensor coordinate frame, and the components of earth rate for the defined coordinate frame. The ability to accurately predict these values will determine the performance limitations of the multisensors. It is recommended that the values of these terms be well established and fitted to polynomials to determine if the multisensor performance can be significantly improved by software modeling these dependent variables.

GLOSSARY

Symbol	Definitions
B_{A2}, B_{B2}	Gyro drift bias for axes A_2 and B_2.
G_1, G_2, G_3	Components of acceleration corresponding to the coordinate frame defined by the multisensor 2.
$GSA_2 * G_1$	Gyro g-sensitive drift due to acceleration along the angular rate axis.
$GSB_2 * G_1$	Cross-axis g-sensitive drift.
$MU_{A2} * G_1$	Gyro g-sensitive drift due to multisensor A-axis acceleration sensitivity.
$MU_{B2} * G_1$	Gyro g-sensitive drift due to multisensor B-axis acceleration sensitivity.
RA_2, RB_2	Pickoff output for gyro axes A_2, B_2.
Wi (i = 1, 2, 3)	Components of earth rate corresponding to coordinate frame defined for multisensor 2.
i (i = 1, 2, 3)	Superscripts 1, 2, 3 represent Collect positions 1, 2, 3.

APPENDIX
DETAILED DISCUSSION

The Collins Phase III multifunction sensor uses a self-calibration scheme to calculate gyro/accelerometer parameters. To accomplish this, the two sensors which make up the multifunction sensor are rotated to three orientations. Figure 1 shows sensor 2 rotation positions. At each position, raw drift rate data is collected for axes A_2 and B_2. The drift rate data is composed of g-sensitive terms, bias terms, and earth rotation terms. It is necessary to determine the values of these gyro parameters. For a particular latitude, the earth rotation terms are known. Therefore, the unknowns reduce to the g-sensitive parameters and the bias parameters.

For each sensor, at each orientation position, two gyro measurement equations can be written in terms of the g-sensitive parameters, the bias parameters, and the earth rate components. Sensor 2 gyro measurement equations for the three collect positions are written as:

(COLLECT POSITION 1)

$$RA_2{}^1 = MU_{B2} * G_3 + B_{A2} - GSA_2 * G_1 + GSB_2 * G_2 - W_1 \quad \text{(A-1)}$$

$$RB_2{}^1 = -MU_{A2} * G_3 + B_{B2} + GSA_2 * G_2 + GSB_2 * G_1 + W_2 \quad \text{(A-2)}$$

(COLLECT POSITION 2)

$$RA_2{}^2 = -MU_{B2} * G_3 + B_{A2} + GSA_2 * G_1 + GSB_2 * G_2 + W_1 \quad \text{(A-3)}$$

$$RB_2{}^2 = MU_{A2} * G_3 + B_{B2} + GSA_2 * G_2 - GSB_2 * G_1 + W_2 \quad \text{(A-4)}$$

(COLLECT POSITION 3)

$$RA_2{}^3 = MU_{B2} * G_1 + B_{A2} + GSA_2 * G_3 + GSB_2 * G_2 + W_3 \quad \text{(A-5)}$$

$$RB_2{}^3 = -MU_{A2} * G_1 + B_{B2} + GSA_2 * G_2 - GSB_2 * G_3 + W_2 \quad \text{(A-6)}$$

At this point, there are six equations and six unknowns. The unknowns are MU_{B2}, MU_{A2}, GSA_2, GSB_2, B_{A2} and B_{B2}. Collins makes the assumption that MU_{B2} and MU_{A2} are small and therefore negligible. This assumption reduces the number of unknowns from six to four with the number of equations staying at six. Therefore, sensor 2 gyro equations for the three collect positions reduce to:

(COLLECT POSITION 1)

$$RA_2^{\,1} = B_{A2} - GSA_2 * G_1 + GSB_2 * G_2 - W_1 \qquad \text{(A-7)}$$

$$RB_2^{\,1} = B_{B2} + GSA_2 * G_2 + GSB_2 * G_1 + W_2 \qquad \text{(A-8)}$$

(COLLECT POSITION 2)

$$RA_2^{\,2} = B_{A2} + GSA_2 * G_1 + GSB_2 * G_2 + W_1 \qquad \text{(A-9)}$$

$$RB_2^{\,2} = B_{B2} + GSA_2 * G_2 - GSB_2 * G_1 + W_2 \qquad \text{(A-10)}$$

(COLLECT POSITION 3)

$$RA_2^{\,3} = B_{A2} + GSA_2 * G_3 + GSB_2 * G_2 + W_3 \qquad \text{(A-11)}$$

$$RB_2^{\,3} = B_{B2} + GSA_2 * G_2 - GSB_2 * G_3 + W_2 \qquad \text{(A-12)}$$

Making the assumption that MU_{B2} and MU_{A2} are zero reduces the unknowns to GSA_2, GSB_2, B_{A2} and B_{B2}. At this point, there are six measurement equations and four unknowns.

The following procedure was used by Collins to solve the four unknown gyro parameters for sensor 2. First, Collins subtracts equation (A-7) from equation (A-9) and solves for GSA_2. Next, Collins subtracts equation (A-10) from equation (A-8) and solves for GSB_2. Collins then substitutes the previously solved parameters for GSA_2 and GSB_2 into equations (A-11) and (A-12) and solves for B_{A2} and B_{B2}, respectively. Collins' solutions for these parameters are written as follows:

$$GSA_2 = \frac{RA_2^{\,2} - RA_2^{\,1} - 2W_1}{2G_1} \qquad \text{(A-13)}$$

$$GSB_2 = \frac{RB_2^{\,1} - RB_2^{\,2}}{2G_1} \qquad \text{(A-14)}$$

$$B_{A2} = RA_2^{\,3} - GSA_2 * G_3 - GSB_2 * G_2 - W_3 \qquad \text{(A-15)}$$

$$B_{B2} = RB_2^{\,3} - GSA_2 * G_2 + GSB_2 * G_3 - W_2 \qquad \text{(A-16)}$$

If MU_{A2} and MU_{B2} are not zero, Collins' solution equations (A-13) through (A-16) would change as follows:

$$GSA_2 = \frac{RA_2{}^2 - RA_2{}^1 - 2W_1}{2G_1} + \frac{MU_{B2} * G_3}{G_1} \qquad \text{(A-17)}$$

$$GSB_2 = \frac{RB_2{}^1 - RB_2{}^2}{2G_1} + \frac{MU_{A2} * G_3}{G_1} \qquad \text{(A-18)}$$

$$B_{A2} = RA_2{}^3 - GSA_2 * G_3 - GSB_2 * G_2 - W_3 - MU_{B2} * G_1 \qquad \text{(A-19)}$$

$$B_{B2} = RB_2{}^3 - GSA_2 * G_2 + GSB_2 * G_3 - W_2 + MU_{A2} * G_1 \qquad \text{(A-20)}$$

Solution equations (A-17) through (A-20) present a problem. Gyro parameters GSA_2 and GSB_2 are now a function of known parameters and unknown parameters (MU_{A2}, MU_{B2}). If Collins' assumption of MU_{B2} and MU_{A2} equals zero is incorrect, then the solution scheme is invalid. In laboratory testing, G_2 and G_3 are very small while G_1 is in the +1g field. Therefore, if MU_{B2} and MU_{A2} are not zero, the only terms that would appear incorrect are the bias terms B_{A2} and B_{B2}.

At this point, it was decided to check the assumption that MU_{B2} and MU_{A2} equal zero. If the assumption was proven invalid, it was decided to show the effect on the gyro parameters.

Before the assumption is made, there are six equations and six unknowns. Mathematically, it is possible to calculate the unknown parameters. Therefore, sensor 2 gyro equations were set up in matrix form (p. 3). Obtaining the augmented matrix (p. 4), and applying row reduction procedures, the solutions to the six gyro parameters were found. The matrix gyro parameter solutions for sensor 2 are:

$$MU_{A2} = \frac{(RB_2{}^1 + RB_2{}^2 - 2RB_2{}^3)\, G_1 + (RB_2{}^2 - RB_2{}^1)\, G_3}{2(G_1{}^2 + G_3{}^2)} \qquad \text{(A-21)}$$

$$MU_{B2} = \frac{(2RA_2{}^3 - RA_2{}^1 - RA_2{}^2 - 2W_3)\, G_1 + (RA_2{}^1 - RA_2{}^2 + 2W_1)\, G_3}{2(G_1{}^2 + G_3{}^2)} \qquad \text{(A-22)}$$

$$GSA_2 = \frac{RA_2{}^2 - RA_2{}^1 - 2W_1}{2G_1} + \frac{MU_{B2} * G_3}{G_1} \qquad \text{(A-23)}$$

$$GSB_2 = \frac{RB_2^{\,1} - RB_2^{\,2}}{2G_1} + \frac{MU_{A2} * G_3}{G_1} \tag{A-24}$$

$$B_{A2} = \frac{(RA_2^{\,1} + RA_2^{\,2})}{2} - GSB_2 * G_2 \tag{A-25}$$

$$B_{B2} = \frac{(RB_2^{\,1} + RB_2^{\,2} - 2W_2)}{2} - GSA_2 * G_2 \tag{A-26}$$

Note that each parameter is a function of that previously solved for parameters and/or known raw data.

To determine whether MU_{A2} and MU_{B2} values were small, self-calibration runs were conducted over a temperature range from -21.5 °F to 140 °F. The data in Table 1 shows that MU_{A2} varies from -37.2 °/hr/g to 23.8 °/hr/g and MU_{B2} varies from -24.3 °/hr/g to 3.6 °/hr/g over the temperature range.

Table 1 also shows the effects of the invalid assumption on B_{A2} and B_{B2}. In laboratory testing, G_1 is approximately equal to +1G. To solve for B_{A2} and B_{B2}, Collins uses equations (A-15) and (A-16), respectively. To include the effects of MU_{A2} and MU_{B2} on the bias terms, equations (A-15) and (A-16) were modified to form equations (A-19) and (A-20), respectively. Using values from Table 1, it can be shown that the modified solutions for B_{A2} and B_{B2} agree with the matrix solution.

Collins uses Collect Position 3 equations (A-11) and (A-12) to solve for B_{A2} and B_{B2}. Collect Position 2 equations (A-9) and (A-10) and Collect Position 1 equations (A-7) and (A-8) could have been used to solve B_{A2} and B_{B2}. It was decided to solve for B_{A2} and B_{B2}, using equations (A-7) through (A-10), and to compare the results to the results Collins calculated using equations (A-11) and (A-12). It was also decided to modify equations (A-7) through (A-12) to take into account for MU_{A2} and MU_{B2}. To simplify the comparison, only the results for B_{A2} will be shown. B_{A2} was solved for using the following equations:

From matrix solution equation (A-25)

$$B_{A2} = \frac{(RA_2^{\,1} + RA_2^{\,2})}{2} - GSB_2 * G_2 \,. \tag{A-27}$$

From equation (A-7), Collect 1

$$B_{A2} = RA_2^{\,1} + GSA_2 * G_1 - GSB_2 * G_2 + W_1 . \tag{A-28}$$

From equation (A-9), Collect 2

$$B_{A2} = RA_2^{\,2} - GSA_2 * G_1 - GSB_2 * G_2 - W_1. \qquad \text{(A-29)}$$

From Equation (A-11), (Collins' solution)

$$B_{A2} = RA_2^{\,3} - GSA_2 * G_3 - GSB_2 * G_2 - W_3. \qquad \text{(A-30)}$$

From equation (A-1)

$$BA2 = RA_2^{\,1} + GSA_2 * G_1 - GSB_2 * G_2 + W_1 - MU_{B2} * G_3. \qquad \text{(A-31)}$$

From equation (A-3)

$$B_{A2} = RA_2^{\,2} - GSA_2 * G_1 - GSB_2 * G_2 - W_1 + MU_{B2} * G_3. \qquad \text{(A-32)}$$

From equation (A-5)

$$B_{A2} = RA_2^{\,3} - GSA_2 * G_3 - GSB_2 * G_2 - W_3 - MU_{B2} * G_1. \qquad \text{(A-33)}$$

Equation (A-27) is the matrix solution for B_{A2}. Equations (A-28) through (A-30) are solutions for B_{A2} from Collins' Collect positions 1, 2, 3, equations. Equations (A-31) through (A-33) are solutions for B_{A2} from Collins' Collect positions 1, 2, 3, modified to take into account for MU_{A2} and MU_{B2}. In laboratory tests, G_1 is approximately equal to 1G while G_2 and G_3 are approximately zero. Therefore, equations (A-31) and (A-32) are approximately the same as equation (A-28) and (A-29), respectively.

Self-calibration tests were performed over a small temperature range. The raw data and gyro parameters were substituted into equations (A-27) through (A-33). Table A-1 shows the results of these self-calibration runs. Notice that all the results for B_{A2} are approximately the same except for B_{A2} solved from equation (A-30). This is the equation Collins uses in their solution scheme. The result for B_{A2} solved from equation (A-33) is Collins'

solution taking into account MU_{A2} and MU_{B2}. This modified solution agrees with the other solutions for B_{A2}. The same comparisons can be shown for B_{B2}.

From the results shown in this appendix, it is concluded that MU_{A2} and MU_{B2} are parameters that need to be calibrated. Neglecting these parameters cause some gyro parameters to be calculated incorrectly. It is recommended that the matrix solutions be used for the gyro parameters of both sensor 1 and sensor 2 of the multifunction sensor.

TABLE A-1. Results of B_{A2} Self-Calibration Runs

B_{A2} Eq.(A-27) (Matrix Solution) °/hr	B_{A2} Eq.(A-28) (from Collect 1) °/hr	B_{A2} Eq.(A-29) (from Collect 2) °/hr	B_{A2} Eq.(A-30) (used by Collins) °/hr	B_{A2} Eq.(A-31) (Modified Eq.(A-28)) °/hr	B_{A2} Eq.(A-32) (Modified Eq.(A-29)) °/hr	B_{A2} Eq.(A-33) (Modified Eq.(A-30)) °/hr
13.9636	13.8142	14.1144	12.0481	13.7852	14.1434	13.9643
10.7605	11.0204	10.5005	16.2857	11.0959	10.5760	10.7651
10.0828	10.4036	9.7619	16.2218	10.4852	9.6804	10.0882
28.8077	28.8351	28.7807	2.7338	28.6930	28.9228	28.8072
28.5187	28.5605	28.4780	2.8734	28.4100	28.6286	28.5186
19.7389	19.6120	19.8657	14.4497	19.5232	19.9544	19.7352
20.1911	20.1481	20.2355	13.8403	20.0434	20.3402	20.1894
19.7615	19.7394	19.7829	13.6698	19.6395	19.8828	19.7591
18.5340	18.5278	18.5400	14.2291	18.3444	18.7233	18.6149
19.4812	19.4754	19.4865	14.7815	19.3996	19.5623	19.4796

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